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COVERED MICROSTRIP LINE WITH GROUND PLANES OF FINITE WIDTH

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Abstract. *Characteristic parameters of a covered microstrip line with ground planes of finite width are determined using hybrid boundary element method (HBEM). This method, developed at the Faculty of Electronic Engineering of Niš is based on the combination of equivalent electrodes method (EEM) and boundary element method (BEM). Results for the characteristic impedance of the observed microstrip line are compared with the corresponding ones obtained by the finite element method.*

Key words: *Characteristic impedance, finite element method (FEM), hybrid boundary element method (HBEM), microstrip line, perfect electric conductor (PEC).*

1. INTRODUCTION

Over the years, many authors have analyzed microstrip lines with finite width dielectric substrate using numerical and analytical methods [1]-[14]. The variational method [5, 7], the boundary element method/method of moments (BEM/MoM) [1], [9]-[11], the conformal mapping and the moving perfect electric wall methods [12]-[13], etc. are some of the commonly used procedures for microstrip lines analysis.

On the other side, the problem of the finite width microstrip ground plane has not been so often researched, although these forms of microstrips are typical in practice. In [4] and [14]-[15] the microstrip line with finite-width dielectric and ground plane was analyzed. A moving perfect electric wall method (MPEW) was applied in [12]. This method is used in combination with the conformal mapping method (CMM). The author obtained simple analytical relations for quasi TEM parameters of microstrip lines. The calculation was performed with the assumption that the conductor thickness is zero.

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In [6] the authors present an efficient numerical technique for characteristic parameters determination of multiconductor transmission lines with homogeneous dielectrics. The influence of finite width ground plane was also investigated. The system of integral equations resulting from the method is solved using Galerkin's method with a pulse approximation. The technique applied in this paper is an improvement of the procedure presented in [8], in the sense of better efficiency and accuracy of the obtained results.

Analysis of structures with ground planes of finite width as well as finite conductor thickness is also possible using the hybrid boundary element method (HBEM) [15]. This method is applied for the microstrip characteristic parameters determination in [15] and [16]. In [17] and [18] the symmetrically coupled microstrip lines with finite and infinite width ground plane are analyzed using the HBEM. Both modes (even and odd) are considered.

Covered coupled microstrip lines parameters are calculated in [19]. The structure that has not been analyzed using HBEM until now is a covered single microstrip line with ground planes of finite width and finite conductor thickness. The analysis of such structure will be presented in this paper. Results obtained for the characteristic impedance will be shown in tables and graphically, as equipotential contours. The main assumption in this analysis involves quasi TEM propagation in the microstrip line.

In order to validate the HBEM values obtained for the characteristic impedance, in terms of accuracy, they have been compared with the corresponding ones obtained by the finite element method (FEM). That method is very useful for application in software for electromagnetic problems solving, including the microwave analysis. Some of this type of software is FEMM [20] or COMSOL [21]. The first one will be applied in this paper for results comparison.

2. Theoretical Background

The HBEM has been applied, until now, for electromagnetic field determination in the vicinity of cable terminations [22], calculation of magnetic force between permanent magnets as well as for microstrip lines parameters determination [23].

A generalization of the HBEM, which is applied in this paper for microstrip lines analysis, was described in detail in [15] and [16]. This method presents a combination of the BEM/MoM, the equivalent electrodes method (EEM) [24] and the point-matching method (PMM).

The main idea of the HBEM is in discretizing each arbitrarily shaped surface of the perfect electric conductor (PEC) electrode as well as an arbitrarily shaped boundary surface between any two dielectric layers. The boundary surfaces are divided into a large number of segments. Each of those segments on PEC electrode is replaced by equivalent electrodes (EEs) placed at their centers. The potential of equivalent electrodes obtained in this manner is the same as the potential of PECs themselves. The segments at any boundary surfaces between the two layers are replaced by discrete equivalent total charges. Those charges are placed in the air [15, 16]. The equivalent electrodes are line charges whose radius is determined in [24].

The Green's function for the electric scalar potential of the charges is used.

Applying the point-matching method (PMM) for the potential of the perfect electric conductor (PEC) electrodes and for the normal component of the electric field at the boundary surface between any two dielectric layers, the system of linear equations is formed.

Increasing the number of the EEs the distances between them becomes smaller. In order to keep stability of the formed system of equations it is necessary that the distances between EEs be larger than their radius. The formed quadratic system of linear equations is well-conditioned. The system matrix always has the greatest values at the main diagonal.

After solving the system of equations, according to [15], it is possible to calculate the capacitance per unit length of the microstrip line, as well as the characteristic impedance and effective relative permittivity.

This method will be described in detail in the following section for characteristic parameters determination of covered microstrip line with the ground planes of finite width and finite conductor thickness.

3. HBEM Application

Geometry of the covered microstrip line, with finite width dielectric substrate placed between two ground planes of finite width, is shown in Fig. 1.

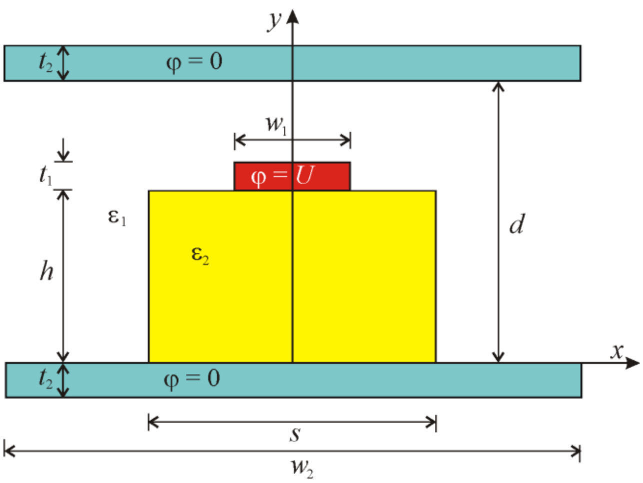

Fig. 1 Problem geometry

The HBEM, based on discretization of boundary surfaces between any two dielectric layers and replacement of those segments with total charges per unit length, is applied.

It should be mentioned that the free surface charges do not exist on boundary surfaces layer 1 - layer 2, so the total surface charges placed between dielectric layers are only surface polarization charges. The equivalent HBEM model is shown in Fig. 2.

- Indices "d", "a" and "t" denote the charges per unit length placed in dielectric ("d") and air ("a") as well as total ("t") charges per unit length, respectively.
- M_i (i = 1,2) is the number of EEs on PECs, with line charges $q'_{\mathrm{d}\,im}$ (m = 1,...,M_i), placed in the layer 2;
- M_j (j = 3,...,5) is the number of EEs on PECs, with line charges $q'_{\mathrm{a}\,jm}$ (m = 1,...,M_j), placed in the layer 1;
- N_i (i = 1,...4) is the number of EEs on boundary surfaces layer 1 – layer 2, with line charges $q'_{\mathrm{t}\,in}$, placed in the air (n = 1,...,N_i);
- $(x_{\mathrm{d}im}, y_{\mathrm{d}im})$, $(x_{\mathrm{a}im}, y_{\mathrm{a}im})$, $(x_{\mathrm{t}in}, y_{\mathrm{t}in})$ are the positions of the EEs.

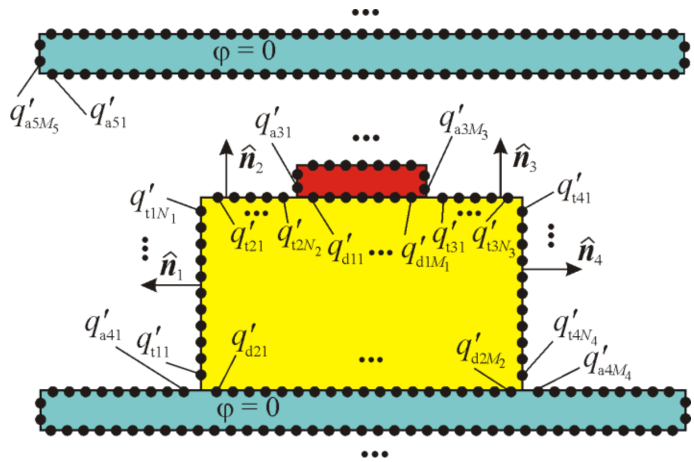

Fig. 2 HBEM model

The electric scalar potential of the system from Fig. 2, is given in Eq. (1).

$$\varphi = \varphi_0 - \sum_{i=1}^{2} \sum_{m=1}^{M_i} \frac{q'_{\mathrm{d}im}}{2\pi\varepsilon_2} \ln\sqrt{(x - x_{\mathrm{d}im})^2 + (y - y_{\mathrm{d}im})^2} - \\ - \sum_{i=3}^{5} \sum_{m=1}^{M_i} \frac{q'_{\mathrm{a}im}}{2\pi\varepsilon_1} \ln\sqrt{(x - x_{\mathrm{a}im})^2 + (y - y_{\mathrm{a}im})^2} - \\ - \sum_{i=1}^{4} \sum_{n=1}^{N_i} \frac{q'_{\mathrm{t}in}}{2\pi\varepsilon_0} \ln\sqrt{(x - x_{\mathrm{t}in})^2 + (y - y_{\mathrm{t}in})^2}, \tag{1}$$

where φ_0 is unknown additive constant, which depends on the chosen referent point for the electric scalar potential.

The procedure for determining the number of unknowns is the following: in order to avoid placing an arbitrary number of unknowns on each boundary surface, an initial parameter N_p is introduced. The number of unknowns is determined as

$$M_1 = \frac{w_1}{h+s} N_{\mathrm{p}}\,,\ M_2 = \frac{s}{h+s} N_{\mathrm{p}}\,,\ M_3 = \frac{w_1 + 2t_1}{h+s} N_{\mathrm{p}}\,,$$

$$M_4 = \frac{w_2 + 2t_2 + 2y}{h+s} N_{\mathrm{p}}\,, \text{ where } y = \frac{w_2 - s}{2}\,,\ M_5 = \frac{2w_2 + 2t_2}{h+s} N_{\mathrm{p}}\,,$$

$$N_1 = N_4 = \frac{h}{h+s} N_{\mathrm{p}}\,,\ N_2 = N_3 = \frac{x}{h+s} N_{\mathrm{p}}\,, \text{ where } x = \frac{s - w_1}{2}\,.$$

The total number of unknowns N_{tot}, will be denoted by:

$$N_{tot} = \sum_{i=1}^{5} M_i + \sum_{i=1}^{4} N_i + 1\,.$$

The electric field is obtained using $\boldsymbol{E} = -\mathrm{grad}(\varphi)$.

A relation between the normal component of the electric field and the total surface charges is given with Eq. (2).

$$\hat{\mathbf{n}}_i \cdot \mathbf{E}_{im}^{(0+)} = \frac{-\varepsilon_2}{\varepsilon_0(\varepsilon_1 - \varepsilon_2)} \eta_{\mathrm{t}im}\,,\ \eta_{\mathrm{t}im} = \frac{q'_{\mathrm{t}im}}{\Delta l_{im}}\,,\ n = 1, \ldots, N_i\,,\ i = 1,2,3,4 \tag{2}$$

where $\hat{\boldsymbol{n}}_i$ ($\hat{\boldsymbol{n}}_1 = -\hat{\boldsymbol{x}}, \hat{\boldsymbol{n}}_2 = \hat{\boldsymbol{y}}, \hat{\boldsymbol{n}}_3 = \hat{\boldsymbol{y}}, \hat{\boldsymbol{n}}_4 = \hat{\boldsymbol{x}}$) are unit normal vectors oriented from the layer ε_2 into layer ε_1.

Applying the procedure described in the previous section, the system of linear equation is formed using the PMM for the potential of the perfect electric conductor given in (1) and the PMM for the normal component of the electric field (2). The unknown free charges per unit length on conductors, and total charges per unit length on the boundary surfaces between two dielectric layers is determined after solving the system of equations.

In order to satisfy the necessary condition of electrical neutrality of the whole covered microstrip line, equation (3) is added:

$$\sum_{i=1}^{2} \sum_{m=1}^{M_i} q'_{\mathrm{d}im} + \sum_{i=3}^{5} \sum_{m=1}^{M_i} q'_{\mathrm{a}im} = 0 \tag{3}$$

In that way, a quadratic system of linear equations is formed. The unknown values are free charges of PECs, total charges per unit length at boundary surfaces between dielectric layers, and unknown additive constant φ_0.

The capacitance per unit length of the observed microstrip line is:

$$C' = \frac{1}{U} \left(\sum_{k=1}^{M_1} q'_{\mathrm{d}1k} + \sum_{k=1}^{M_3} q'_{\mathrm{a}3k} \right) \tag{4}$$

The characteristic impedance is given in (5)

$$Z_{\mathrm{c}} = Z_{\mathrm{c}0} / \sqrt{\varepsilon_r^{eff}}\,, \tag{5}$$

where $\varepsilon_r^{eff} = C'/C'_0$ is the effective relative permittivity of the microstrip line, and Z_{c0} is the characteristic impedance of the microstrip line placed in the air. Also, with C'_0 the capacitance per unit length of the microstrip line without dielectrics (free space) is denoted.

In order to validate and compare the obtained results for the characteristic impedance, the software FEMM [20] is used.

4. Numerical Results

A computer code based on the procedure described in previous section, is written in Mathematica [25]. All calculations were performed on computer with dual core INTEL processor 2.8 GHz and 4 GB of RAM.

The results convergence and the computation time are shown in Table 1. The values of the effective relative permittivity, the characteristic impedance are determined for: $\varepsilon_{r1} = 1$, $\varepsilon_{r2} = 3$, $w_1/d = 1.0$, $t_1/w_1 = 0.05$, $w_2/d = 3.0$, $t_2/w_2 = 0.1$, $h/d = 0.5$ and $s/d = 2.0$.

Table 1 Convergence of the results and computation time

N_p	N_{tot}	ε_r^{eff}	$Z_c[\Omega]$	$t(s)$
5	66	1.7008	44.665	0.3
10	98	1.8648	42.234	0.4
15	134	1.7107	44.228	0.7
20	166	1.7825	43.544	1.0
50	376	1.8559	43.328	4.5
75	550	1.8707	43.343	9.6
85	618	1.8744	43.346	12.1
100	722	1.8786	43.349	16.5
125	894	1.8836	43.350	25.2
135	964	1.8846	43.356	29.3
150	1068	1.8866	43.355	36.0
160	1136	1.8877	43.355	41.3
170	1242	1.8887	43.356	49.3
200	1414	1.8908	43.356	64.5
250	1760	1.8935	43.356	100.5
300	2106	1.8953	43.356	143.2
325	2278	1.8963	43.356	171.0

The "computation time" is the time spent for determining the number of unknowns, their positioning, forming a matrix elements, solving the system of equations, the characteristic parameters calculation. Most of the calculation time is spent on matrix fill. For example, when the $N_{\text{tot}} = 1068$, the time for determining the number of unknowns and their positioning is 0.2 s and for the matrix fill 32 s. For solving the system of linear equation is spent 3.3 s and for the capacitance, characteristic impedance and effective dielectric permittivity calculation 0.5 s.

From Table 1 is evident that a good convergence of the results is achieved in a short computation time. Sufficient accuracy is obtained for 1242 unknowns, so there is no need to increase the number of EEs.

Equipotential contours are shown in Fig. 3, for: N_p = 150, ε_{r1} = 1, ε_{r2} = 3, w_1/d = 1.0, t_1/w_1 = 0.05, w_2/d = 3.0, t_2/w_2 = 0.1, h/d = 0.5 and s/d = 2.0.

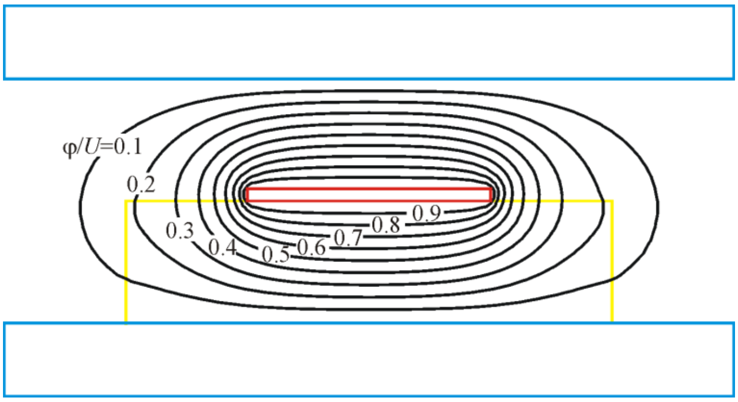

Fig. 3 Equipotential contours

In order to verify the obtained HBEM values, a comparison of HBEM and FEMM results for the effective dielectric permittivity and the characteristic impedance versus h/d is given in Table 2.

The discrepancy of these results is less than 0.6 %. It should be mentioned that the classical comparison of results does not make sense here. These methods (HBEM and FEM) are applied under different conditions. The number of unknowns in the HBEM application was about 1100. On the other hand, the corresponding FEMM model was created with a few thousand finite elements. Increasing the number of finite elements, accuracy of FEMM increases too, so it is possible to “compare” and verify the HBEM results.

Table 2 Verification of results for effective dielectric permittivity and characteristic impedance of microstrip line versus h/d for parameters: N_p = 150, ε_{r1} = 1, ε_{r2} = 3, w_1/d = 1.0, t_1/w_1 = 0.05, w_2/d = 3.0, t_2/w_2 = 0.1 and s/d =2.0

	HBEM		FEM	
h/d	ε_r^{eff}	$Z_c[\Omega]$	ε_r^{eff}	$Z_c[\Omega]$
0.2	2.3745	27.897	2.3740	28.056
0.3	2.2007	35.847	2.2089	35.895
0.4	2.0424	40.946	2.0568	40.883
0.5	1.8866	43.355	1.9068	43.208
0.6	1.7286	42.904	1.7530	42.702
0.7	1.5588	39.030	1.5898	38.812
0.8	1.3685	30.462	1.4072	30.320

Distributions of characteristic impedance versus different parameters are shown in the following figures. Fig. 4 shows the influence of ground plane thickness on the characteristic impedance of microstrip line.

The input data are: $N_p = 150$, $\varepsilon_{r1} = 1$, $\varepsilon_{r2} = 3$, $w_1/d = 1.0$, $t_1/w_1 = 0.05$, $w_2/d = 3.0$ and $s/d = 2.0$.

From this figure it is evident that for corresponding input data, the characteristic impedance does not depend on the ground planes thickness. The characteristic impedance depends on the conductor's distance from the planes (parameter h/d). Increasing this parameter, the characteristic impedance first increases, and then decreases. The maximum value is when the conductor is equidistant from the ground planes.

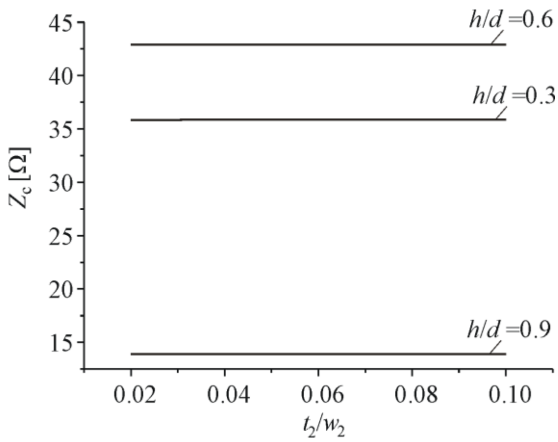

Fig. 4 Distribution of characteristic impedance versus t_2/w_2 for different values of parameter h/d

Distribution of characteristic impedance versus w_1/d and s/d is shown in Fig. 5. Also, there are given values for characteristic impedance of microstrip line with parallel ground planes of infinite width [7].

The influence of dielectric substrate width as well as ground planes width is given in Fig. 6. Increasing the substrate width, the characteristic impedance decreases. The influence of planes width on characteristic impedance exists, but it can be neglected.

Increasing the substrate height, the characteristic impedance first increases first, then decreases as the conductor approaches the upper plane, Fig. 7. The dielectric permittivity of substrate has also the influence on the characteristic impedance value. Increasing the substrate permittivity, the characteristic impedance values decrease.

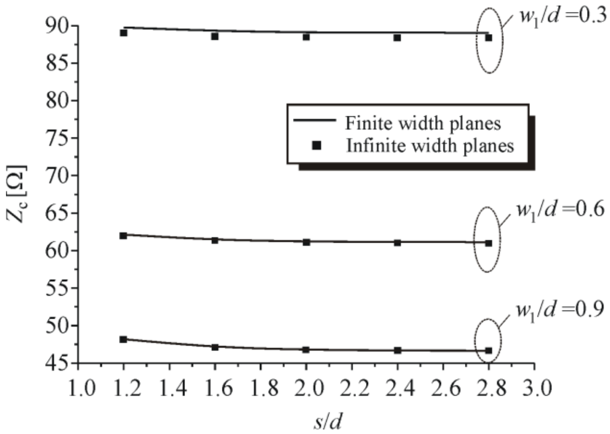

Fig. 5 Distribution of characteristic impedance versus *s*/*d* for different values of parameter w_1/d

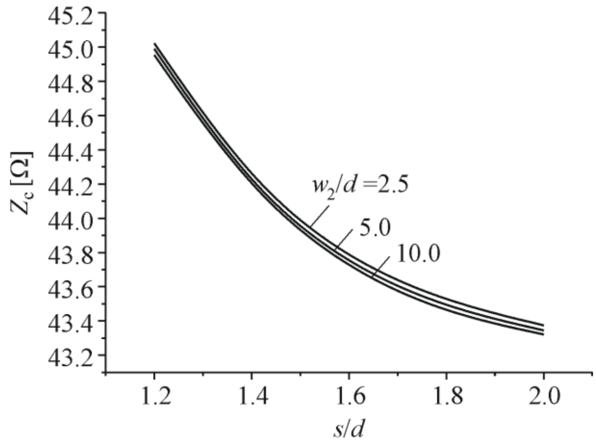

Fig. 6 Distribution of characteristic impedance versus *s*/*d* for different values of parameter w_2/d

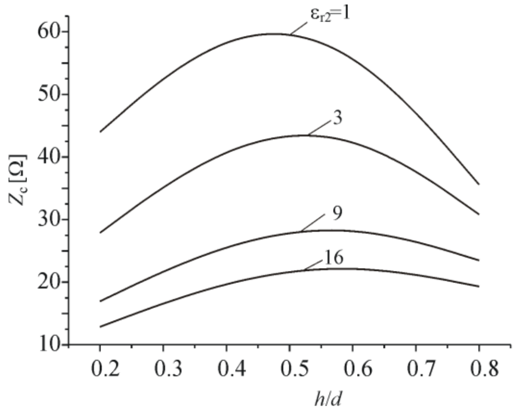

Fig. 7 Distribution of characteristic impedance versus h/d for different values of parameter ε_{r2}

Distribution of polarization charges per unit length along boundary surface is shown in Fig. 8.

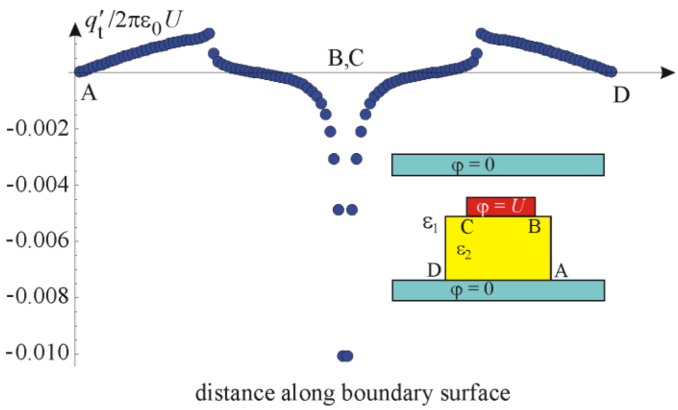

Fig. 8 Distribution of polarization charges per unit length along boundary surface

5. Conclusion

The aim of this paper is to apply a very efficient HBEM, based on a combination of EEM and BEM, for determining the characteristic impedance of the covered microstrip line with ground planes of finite width. That configuration has not been analyzed so far using HBEM.

The quasi TEM analysis is applied. The main advantage of this method is the possibility to solve arbitrarily shaped, multilayered configuration of microstrip lines, with finite dimension of ground planes and conductor thickness, without any numerical integration. Of course, there are other methods that can analyze this structure, but the HBEM is simple and accurate procedure. The convergence of the results is good and the computation time is very short.

The analysis of this microstrip was performed for different values of microstrip parameters. The influence of permittivity of layer 2 on the characteristic impedance is evident. Also, the results show that for $w_2/w_1 > 2.5$ the influence of finite width of ground planes on the characteristic impedance values can be neglected.

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